

The Dark Energy Scale in Superconductors: Innovative Theoretical and Experimental Concepts

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Abstract

We revisit the cosmological constant problem using the viewpoint that the observed value of dark energy density in the universe actually represents a rather natural value arising as the geometric mean of two vacuum energy densities, one being extremely large and the other one being extremely small. The corresponding mean energy scale is the Planck Einstein scale $l_{PE} = \sqrt{l_P l_E} = (\hbar G/c^3 \Lambda)^{1/4} \sim 0.037 mm$, a natural scale both for dark energy and the physics of superconductors. We deal with the statistics of quantum fluctuations underlying dark energy in superconductors and consider a scale transformation from the Planck scale to the Planck-Einstein scale which leaves the quantum physics invariant. Our approach unifies various experimentally confirmed or conjectured effects in superconductors into a common framework: Cutoff of vacuum fluctuation spectra, formation of Tao balls, anomalous gravitomagnetic fields, non-classical inertia, and time uncertainties in radioactive superconductors. We propose several new experiments which may further elucidate the role of the Planck-Einstein scale in superconductors.

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1 Introduction

Observational cosmology of the last decade has revealed a composition of the universe that poses one of the greatest challenges theoretical physics has ever faced: We are living in a universe that exhibits accelerating expansion in (approximately) four space-time dimensions, with a de Sitter spacetime metric with cosmological constant $\Lambda = 1.29 \times 10^{-52}[m^{-2}]$ [1, 2, 3, 4, 5]. A small cosmological constant is equivalent to a small vacuum energy density (dark energy density with equation of state $w = -1$) given by

$$\rho_{vac\Lambda} = \frac{c^4\Lambda}{8\pi G} = 6.21 \times 10^{-10}[J/m^3]. \quad (1)$$

In quantum field theory, the cosmological constant counts the degrees of freedom of the vacuum. Heuristically, we may sum the zero-point energies of harmonic oscillators and write:

$$E_{vac} = \sum_i \left(\frac{1}{2} \hbar \omega_i \right) \quad (2)$$

The sum is manifestly divergent. The natural maximum cutoff to impose on a quantum theory of gravity is the Planck frequency, leading to the Planck energy density of the vacuum: $\rho_P = E_P/l_P^3 = \frac{c^7}{G^2\hbar} = 4.6 \times 10^{113}[J/m^3]$. This prescription yields an ultraviolet enumeration of the zero-point energy that is 122 orders of magnitude above the measured value, eq.(1). This is the well-known cosmological constant problem. There is also a natural minimum cutoff that we can impose on the sum eq.(2), which is the minimum energy, $E_E = \sqrt{c^2\hbar^2\Lambda}$ associated with the cosmological length scale (or Einstein length scale), $l_E := \Lambda^{-1/2}$. This leads to the Einstein energy density for the vacuum, $\rho_E = c\hbar\Lambda^2 = 5.26 \times 10^{-130}[J/m^3]$, which is 121 orders of magnitude below the measured value of the dark energy density, eq.(1). Apparently, the correct value of vacuum energy density in the universe as seen from WMAP [1] observations is approximately the geometric mean of both values (see also [6, 7, 8] for related work).

We may therefore ask: What is the domain of physics and phenomenology that provides a natural scale for dark energy? In fact, the observed dark energy scale is very similar to typical energy scales that occur in solid state physics in the physics of superconductors. The principal idea discussed in recent papers [9, 10, 11] is that this is not a random coincidence, but that there is a deep physical reason for this coming from a similar structure of the theory of superconductors and that of dark energy. A possible model in this direction is the Ginzburg-Landau theory of dark energy as developed

in [10]. In this model vacuum fluctuations exhibit a phase transition from a gravitationally active to a gravitationally inactive state at a critical frequency, similar to the superconductive phase transition at a critical temperature. On the experimental side, there is an interesting possibility arising out of this approach, namely that superconductors could be used as suitable detectors for quantum fluctuations underlying dark energy [10, 11]. In the following we will further work out this concept. In particular, we will deal with uncertainty relations for vacuum energy densities and space-time volumes, and deal with the corresponding fluctuation statistics in superconductors. We will suggest several new laboratory experiments which could be performed to test the theoretical concepts.

We will re-investigate the cosmological constant problem by considering an uncertainty relation between vacuum energy density and the four-dimensional volume of the universe. We start from the Einstein-Hilbert action and show that in this approach one actually obtains an ‘inverse’ cosmological constant problem: The cosmological constant comes out *too small* by 120 orders of magnitude! This naturally suggests to regard the observed dark energy density of the universe as the geometric mean of two values of vacuum energy, one being 120 orders of magnitude too large and the other one being 120 orders of magnitude too small. The corresponding mean energy scale is the Planck-Einstein scale, corresponding to lengths of about 0.037mm, a natural scale both for dark energy and superconductive materials.

Quite interesting phenomena may arise out of the fact that the relevant length scale for quantum fluctuations is the Planck-Einstein scale in superconductive materials. Basically, our model for quantum fluctuations in the superconductor is like a model of quantum gravity where the Planck mass $m_P = (\hbar c/G)^{1/2}$ is replaced by a much smaller value, the Planck-Einstein mass $m_{PE} = (\hbar^3 \Lambda / cG)^{1/4}$. Formally replacing the Planck mass by much smaller values has also been discussed in the context of extra dimensions [12]. Our approach here does not require extra dimensions but just a superconducting environment. We will analyse the observed formation of large spherical clusters of superconductive particles, so-called Tao balls [13, 14, 15, 16] in this context, a phenomenon that is so far unexplained in the usual theory of superconductors, but which can be understood using our current approach. We will also suggest to measure the formal fluctuations of space-time in the superconductor by looking at the lifetime of radioactive superconductors, as well as by comparing two clocks, one located inside the superconductive cavity, and the other being located outside.

This paper is organized as follows. In section 2 we sketch the inverse cosmological problem and describe how to obtain a natural scale of dark energy, the Planck-Einstein scale. Properties of this scale are summarized in section

3. In section 4 we deal with scale transformations from the Planck scale (relevant in a non-superconducting environment) to the Planck-Einstein scale (relevant in a superconducting environment). We discuss several interesting phenomena that may produce measurable effects in this context: Cutoffs of quantum noise spectra, formation of Tao balls, fundamental time uncertainties in radioactive superconductors and non-classical inertia. A more detailed model for the formation of Tao balls is discussed in section 5. Finally, in section 6 we list some suggestions for future experiments that could further clarify the role of dark energy and gravity in superconductors.

2 Inverse Cosmological Constant Problem and the Uncertainty Principle

The cosmological constant problem is the problem that the typical value of vacuum energy density predicted by quantum field theory is *too large* by a factor of 10^{120} as compared to astronomical observations. Here we show that using a different argument, one can actually get a value that is *too small* by a factor of order 10^{-120} . Hence the observed value of the cosmological constant in nature seems not that unnatural at all, being the geometric mean of both values.

We start from the Einstein Hilbert action

$$S = \int [k(R - 2\Lambda) + L_M] \sqrt{-g} d^4x, \quad (3)$$

where R is the Ricci scalar, g is the determinant of the space-time Lorentz metric, L_M is the matter Lagrangian, and the constant k is

$$k = \frac{c^4}{16\pi G}, \quad (4)$$

where G is the gravitational constant. Einstein's field equations are invariant under complex transformations of the space-time coordinates $x^\mu \rightarrow iy^\mu$ except for the cosmological constant term [17], which is the only relevant term for our approach in the following. The part S_Λ of the action corresponding to the cosmological constant Λ can be written as a product between the vacuum energy density $\rho_{vac\Lambda}[J/m^3]$ and the four-dimensional volume $V[m^4]$

$$S_\Lambda = -\rho_{vac\Lambda} V, \quad (5)$$

where the four-dimensional volume is expressed in its covariant form as

$$V = \int d^4x \sqrt{-g}, \quad (6)$$

and the vacuum energy density is given by

$$\rho_{vac\Lambda} = \frac{c^4\Lambda}{8\pi G}. \quad (7)$$

We may now regard $\rho_{vac\Lambda}$ and V occurring in eq. (5) as canonically conjugated quantities, as previously suggested in [18, 19]. In a quantum theory of gravity, we expect that the fluctuations in one observable are related to fluctuations in its conjugate, according to Heisenberg's uncertainty relation. Thus

$$\Delta\rho_{vac\Lambda}\Delta V \sim \hbar c. \quad (8)$$

This resembles a kind of uncertainty relation in 4-dimensional spacetime. Substituting eq.(7) into eq.(8), we can write eq.(8) as

$$\Delta\Lambda\Delta V \sim 8\pi l_P^2 \quad (9)$$

where $l_P = \sqrt{G\hbar/c^3} = 1.61 \times 10^{-35}[m]$ is the Planck length.

Let us now assume that the universe has a finite lifetime τ . τ is bounded from below by the current age of the universe. From the measured Hubble constant, $H_0 = 2.3 \times 10^{-18}[s^{-1}]$, which is of the order of the inverse age of the universe, the four-volume of the universe can be estimated:

$$\Delta V \sim V \sim \frac{4}{3}\pi(c\tau)^4 \sim \frac{4}{3}\pi\left(\frac{c}{H_0}\right)^4 \sim 1.2 \times 10^{105}[m^4] \quad (10)$$

Substituting eq.(10) into eq.(9) a typical order of magnitude of the cosmological constant can be computed, regarding the present value as a quantum fluctuation:

$$\Delta\Lambda \sim \Lambda = 5.44 \times 10^{-174}[1/m^2] \quad (11)$$

This value is in total disagreement with the experimental results, being 121 orders of magnitude *below* the value measured by WMAP:

$$\Lambda = 1.29 \times 10^{-52}[1/m^2] \quad (12)$$

Apparently we get by this formal approach a different cosmological constant problem, which we may call the *inverse* (or infrared) cosmological problem. The typical order of magnitude of the cosmological constant, as derived from this formal approach, turns out to be 120 orders of magnitude *too small* as compared to the astronomical observations. If the universe lives much longer, the estimated value of the typical order of magnitude of Λ would even further decrease.

Our conclusion is that the observed value of the cosmological constant is not so unnatural after all. It's just given by the geometric mean of both

approaches, the one starting from zeropoint energies in quantum field theories and the one starting from an uncertainty relation for S_Λ . Full symmetry between both approaches is obtained if $\tau \sim H_0^{-1}$.

Let us now try to reconcile both approaches. Following Sorkin's work [20, 21] in causal set theory, fluctuations in Λ are inversely related to fluctuations in V . The fluctuations of relevance to us are in the number n_{cells} of Planck sized cells that fill up the four-dimensional spacetime of the universe:

$$n_{cells} \sim \frac{V}{l_P^4} \Rightarrow \Delta n_{cells} \sim \sqrt{n_{cells}} \Rightarrow \Delta V \sim \sqrt{V} l_P^2, \quad (13)$$

Substituting eq.(13) into eq.(9), we obtain:

$$\Delta \Lambda \sqrt{V} \sim 8\pi \quad (14)$$

Substituting the value of the four-volume of the universe of eq.(10) into eq.(14) we find a value of the cosmological constant in agreement with the experimentally measured value eq.(12):

$$\Delta \Lambda \sim \Lambda \sim 10^{-52} [1/m^2] \quad (15)$$

In summary we see that the quantization of the universe's spacetime volume with Planck sized four-dimensional cells can solve the cosmological constant problem if we interpret the value of cosmological constant as being due to statistical fluctuations of the total number of cells making up this volume, according to the uncertainty principle eq.(9).

3 The Planck-Einstein Scale

The Planck-Einstein scale corresponds to the geometric mean value between the Planck scale, l_P , which determines the highest possible energy density in the universe, and the cosmological length scale, or Einstein scale, $l_E = \Lambda^{-1/2}$, which determines the lowest possible energy in the universe [22, 23]. The Planck-Einstein energy density is the geometric mean $\rho_{PE} = \sqrt{\rho_P \rho_E}$ between the two energy densities, and the Planck-Einstein length $l_{PE} = \sqrt{l_P l_E}$ is the geometric mean of the two length scales in the universe [6, 7, 8]. In the following table we list side by side the relevant quantities and their numerical values:

	Einstein scale	Planck-Einstein Scale	Planck scale
	Λ, \hbar, c, k	$\Lambda, \hbar, c, k G$	c, \hbar, k, G
Temperature [K]	$T_E = \frac{1}{k} \sqrt{c^2 \hbar^2 \Lambda}$	$T_{PE} = \sqrt{T_E T_P}$	$T_P = \frac{1}{k} \sqrt{\frac{\hbar c^5}{G}}$
	2.95×10^{-55}	60.71	1.42×10^{32}
Time [s]	$t_E = \sqrt{\frac{1}{c^2 \Lambda}}$	$t_{PE} = \sqrt{t_E t_P}$	$t_P = \sqrt{\frac{\hbar G}{c^5}}$
	2.58×10^{43}	1.26×10^{-13}	5.38×10^{-44}
Length [m]	$l_E = \sqrt{\frac{1}{\Lambda}}$	$l_{PE} = \sqrt{l_E l_P}$	$l_P = \sqrt{\frac{\hbar G}{c^3}}$
	8.8×10^{25}	3.77×10^{-5}	1.61×10^{-35}
Mass [Kg]	$m_E = \sqrt{\frac{\hbar^2 \Lambda}{c^2}}$	$m_{PE} = \sqrt{M_E M_P}$	$m_P = \sqrt{\frac{\hbar c}{G}}$
	5.53×10^{-95}	9.32×10^{-39}	2.17×10^{-8}
Energy [J]	$E_E = \sqrt{c^2 \hbar^2 \Lambda}$	$E_{PE} = \sqrt{E_E E_P}$	$E_P = \sqrt{\frac{\hbar c^5}{G}}$
	4.07×10^{-78}	8.38×10^{-22}	1.96×10^9
Energy density [J/m^3]	$\rho_E = \sqrt{c^2 \hbar^2 \Lambda^4}$	$\rho_{PE} = \sqrt{\rho_E \rho_P}$	$\rho_P = \sqrt{\frac{c^{14}}{G^4 \hbar^2}}$
	5.26×10^{-130}	3.73×10^{-9}	4.6×10^{113}

Explicitly one has the following formulas at the Planck-Einstein scale:

$$E_{PE} = k T_{PE} = \left(\frac{c^7 \hbar^3 \Lambda}{G} \right)^{1/4} = 5.25 [meV] \quad (16)$$

$$m_{PE} = \frac{E_{PE}}{c^2} = \left(\frac{\hbar^3 \Lambda}{c G} \right)^{1/4} = 9.32 \times 10^{-39} [Kg] \quad (17)$$

$$l_{PE} = \frac{\hbar}{M_{PE} c} = \left(\frac{\hbar G}{c^3 \Lambda} \right)^{1/4} = 0.037 [mm] \quad (18)$$

$$t_{PE} = \frac{l_{PE}}{c} = \left(\frac{\hbar G}{c^7 \Lambda} \right)^{1/4} = 1.26 \times 10^{-13} [s] \quad (19)$$

$$\rho_{PE} = \frac{E_{PE}}{l_{PE}^3} = \frac{c^4 \Lambda}{G} = 104 [eV/mm^3] \quad (20)$$

One readily notices that the numerical values of Planck-Einstein quantities correspond to typical time, length or energy scales in superconductor physics, as well as to typical energy scales for dark energy. In previous papers it has been pointed out [9, 10, 11] that there could be a deeper reason for this coincidence: It is possible to construct theories of dark energy that bear striking similarities with the physics of superconductors. In these theories the Planck-Einstein scale replaces the Planck scale as a suitable cutoff for vacuum fluctuations.

4 Scale transformation in superconductors

Our main hypothesis in this paper, to be worked out in the following, is that when proceeding from a normal to a superconducting environment it makes sense to consider a scale transformation from the Planck length to the Planck-Einstein length, which keeps many features of the quantum physics invariant. We will give many examples below. The scale transformation may induce new interesting observable phenomena in superconductors. Quantum gravity phenomena that normally happen at the Planck scale l_P only could possibly induce related phenomena at the Planck-Einstein scale in a superconducting environment. Due to our scale transformation, the gravitational constant $G = \hbar c/m_P^2$ formally becomes much stronger in a superconductor if m_P is replaced by the much smaller value m_{PE} . Similarly, frame dragging effects and gravitomagnetic fields could become much stronger as well, in line with recent experimental observations [24, 25, 26]. The vacuum energy density of vacuum fluctuations would become much smaller as well (of the order of dark energy density). In the following subsections we will investigate the consequences of our scale transformation hypothesis and show that the hypothesis is consistent with some recent experimental observations for superconducting materials. Moreover, we will predict some new phenomena as a consequence of the scale transformation that could be experimentally tested.

4.1 Cutoff for vacuum fluctuations in superconductors

As mentioned before, in quantum field theories the natural cutoff frequency ω_c for vacuum fluctuations is given by $\hbar\omega_c \sim m_P c^2$. This leads to the cosmological constant problem, since the corresponding vacuum energy obtained by integrating over all frequencies up to ω_c is much too large. In Josephson junctions (two superconductors separated by a thin insulator) vacuum fluctuations of the electromagnetic field can lead to measurable noise spectra [27, 28]. This measurability is due to the Josephson effect and the fluctuation dissipation theorem (details in [9, 29]). However, the maximum Josephson frequency that can be reached with a given superconductor (and thus the cutoff frequency of measurable vacuum fluctuations) is determined by the gap energy of the superconductor. This gap energy of a superconductor is proportional to kT_c , where T_c is the critical temperature of the superconductor under consideration (in the BCS theory, the proportionality factor is given by 3.5). Thus measurable noise spectra induced by vacuum fluctuations can only be measured in superconducting Josephson junctions up to a critical value of the order $\hbar\omega_c \sim kT_c$.

Re-interpreted in terms of our scale transformation, this means that for superconductors the Planck scale m_P as a cutoff for vacuum fluctuations is formally replaced by something of the order of the Planck-Einstein scale, since the critical temperature $T_c = 1...140K$ of normal and high- T_c superconductors is of the same order of magnitude as the Planck-Einstein temperature $T_{PE} = 60.7K$. Hence our scale transformation hypothesis makes sense for vacuum fluctuations and vacuum energy as observed in a superconducting environment. The relevant scale factor is of the order $l_{PE}/l_P \sim 10^{30}$.

4.2 Formation of Tao balls

When a strong electric field is applied to a mixture of superconducting and non-superconducting particles, a remarkable effect is observed [13, 14, 15, 16]: Millions of superconducting microparticles of μm size spontaneously aggregate into spherical balls of mm size. The normal particles in the mixture do not show this behavior, only the superconducting ones. The effect has not been explained within the conventional theory of superconductors so far. In fact, within the conventional theory of superconductors one expects that normal particles respond to electrostatic fields in just the same way as superconducting ones do. Hence the Tao effect represents an unsolved puzzle: Superconducting and non-superconducting matter behave in a fundamentally different way. Assuming that the superconducting and non-superconducting particles differ in no other way, one may even regard this effect as pointing towards a violation of the equivalence principle, or more generally the principle of general covariance.

In a sense the formation of Tao balls reminds us of a kind of ‘planet formation’ on a scale that is much smaller than the solar system, which is possible for superconducting matter only. When working out this analogy, again relevant scale factors of the order $10^{30} \sim l_{PE}/l_P \sim l_E/l_{PE}$ arise: Tao balls have a size of order $10^{-3}m$, whereas typical planets such as the earth have a size of order $l_E \sim 10^7m$. Hence the volume of a Tao ball is smaller than the volume of a typical planet by a factor 10^{-30} , and so is the mass of the Tao ball. The possible role of gravitational forces in the formation process of Tao balls is further discussed in section 5.

4.3 Fundamental space-time uncertainty in a radioactive superconductor

In the following we predict a new effect for radioactive superconductors, which arises out of the scale transformation and which could possibly be

confirmed in future experiments. We start from an effective Planck length comparable to the Planck-Einstein length in superconductors. We are lead to envisage that the spacetime volume of a superconductor is made of Planck-Einstein sized cells, l_{PE}^4 , which will statistically fluctuate according to eq.(13):

$$\Delta V \sim \sqrt{V} l_{PE}^2 \quad (21)$$

What should we now take for the space-time volume of a superconductor? The problem is well-defined if we consider a superconducting material with a finite life time, a radioactive superconductor. Let v denote the volume of the superconducting material and τ the mean life time of the radioactive material. We then choose the 4-volume as

$$V = v c \tau. \quad (22)$$

This is similar to the approach in section 2, where we considered a universe with a finite life time τ of order H_0^{-1} . Since the 3-volume v is fixed, for a superconductor in the laboratory there can only be a time uncertainty Δt given by

$$\Delta V = v c \Delta t. \quad (23)$$

Putting eq. (22) and (23) into (21) we obtain an equation for the order of magnitude of a fundamental time uncertainty in radioactive superconductors:

$$\Delta t \sim \sqrt{\frac{\tau}{c v}} l_{PE}^2 \quad (24)$$

To estimate some numbers, let us consider the metastable state of a Nb^{90m} superconductor with a mean life time of $\tau = 34.6[s]$. The volume of the superconductor in Tate's experiment [30] (just as an example) is $v = 1.28 \times 10^{-13}[m^3]$. From this we get

$$\Delta t \sim 1.3 \times 10^{-6}[s]. \quad (25)$$

Fundamental time uncertainties of the above kind should create a broadening of the decay energy line width $\Gamma = \hbar/\tau$:

$$\frac{\Delta t}{\tau} \sim \frac{\Delta \Gamma}{\Gamma}. \quad (26)$$

For the above example we get

$$\frac{\Delta \Gamma}{\Gamma} \sim 10^{-8} \quad (27)$$

which is challenging to measure. The smallness of the above number might explain why this effect has not been revealed by the experiments of Mazaki [31] on the search for a superconducting effect on the decay of Technetium-99m. However the possibility of time fluctuations in radioactive superconductors offers a new perspective to interpret the positive results from Olin [32] on the influence of superconductivity on the lifetime of Niobium-90m.

4.4 Uncertainty principle and non-classical inertia in superconductors

Tajmar et al. [24, 25, 26] have measured anomalous acceleration signals around isolated accelerated superconductors, as well as anomalous gyroscope signals around constantly rotating superconductors. These signals can be interpreted in terms of an anomalous gravitomagnetic field that is about 30 orders of magnitude larger than expected from normal gravity. We note again that $l_{PE}/l_P \sim l_E/l_{PE} \sim 10^{30}$, thus the effect could again stand in relation to a scale transformation in superconductors.

A recent paper [11] connects the anomalous gravitomagnetic fields and non-classical inertial properties of superconductive cavities [33, 34] with the electromagnetic model of dark energy of Beck and Mackey [10]. In this approach the vacuum energy stored in a given superconductor is given by

$$\rho_{vac} = \frac{\pi(\ln 3)^4}{2} \frac{k^4}{(ch)^3} T_c^4 \quad (28)$$

One defines an dimensionless parameter χ by

$$\chi = \frac{B_g}{\omega} = -2 \frac{g}{a} \quad (29)$$

Here B_g is the gravitomagnetic field created by a rotating superconductor, ω the angular velocity of the rotating superconductor, g is the acceleration measured inside the superconductive cavity and a the acceleration communicated to the superconductive cavity [35, 36]: For a cavity made of normal matter $\chi = 2$, which means that the gravitational Larmor theorem, $B_g = 2\omega$ [37], and the principle of general covariance, $g = -a$, are verified. For a superconductive cavity χ turns out to be a function of the ratio between the electromagnetic vacuum energy density contained in the superconductor, ρ_{vac} as given by eq.(28), and the cosmological vacuum energy density, $\rho_{vac\Lambda}$ as given by eq.(7):

$$\chi = \frac{3}{2} \frac{\rho_{vac}}{\rho_{vac\Lambda}} \quad (30)$$

Substituting eq.(28) and eq.(21) into eq.(8) we obtain for the typical size of fluctuations in χ the value

$$\Delta\chi\sqrt{V} \sim \frac{2\pi^2}{3}l_{PE}^2. \quad (31)$$

This can be interpreted in the sense that the inertia inside a superconductive cavity change with respect to their classical laws due to the superconductor's spacetime volume fluctuations. Again the relevant length scale is the Planck-Einstein length, rather than the Planck length.

5 Gravitational surface tension of Tao balls

As already mentioned in section 4.2, a Tao ball is made up of many superconductive microparticles of size $r \sim 1\mu m$ and its radius is of the order $a \sim 1mm$. Therefore a Tao Ball consists of roughly

$$\left(\frac{a}{r}\right)^3 \sim 10^6 microparticles. \quad (32)$$

Tao *et al.* [13] proposed to explain the strong cohesion into spherical balls by a new type of surface tension σ . Also Hirsch [16] emphasizes that the formation of Tao balls cannot be understood by conventional superconductor physics.

The Tao ball surface energy ϵ_a is the product of the Tao ball surface $4\pi a^2$ and the surface tension σ :

$$\epsilon_a = 4\pi a^2 \sigma \quad (33)$$

If there are $(a/r)^3$ separated spherical particles then the total surface energy ϵ_{tot} is

$$\epsilon_{tot} = 4\pi r^2 \sigma \left(\frac{a}{r}\right)^3 \sim \epsilon_a \frac{a}{r} \sim 100\epsilon_a. \quad (34)$$

Therefore the surface energy ϵ_a of a Tao ball is just 1 % of the total surface energy ϵ_{tot} of the separated superconductive microparticles it consists of. This makes it plausible why macroscopic objects form. However, the question is what type of force creates the surface tension. It must be a force that is strong for superconducting matter only, and it must allow for the spherical symmetry of the objects formed.

Let us consider a homogeneous spherical body of mass m , uniform density ρ , and radius a , generating a Newtonian gravitational field. We may formally define a gravitational surface energy ϵ_g (also called surface pressure) [38] by

$$\epsilon_g = G \frac{m^2}{a} \quad (35)$$

where G is the gravitational constant. The gravitational surface tension σ_g is then given by

$$\epsilon_g = 4\pi a^2 \sigma_g, \quad (36)$$

or

$$\sigma_g = \frac{1}{3} G m \rho = \frac{m g_0}{4\pi a}, \quad (37)$$

where $g_0 = Gm/a^2$ is the gravitational acceleration at the surface of the body. Is it consistent to explain the strong cohesion of Tao balls via a gravitational surface tension of the type of eq.(37)?

To answer this question let us assume that the gravitational acceleration g_0 responsible for this hypothetical surface tension σ_g , eq.(37), is generated from the acceleration A communicated to the Tao ball by the electric field E_0 via its electric charge $|q|$, according to the law of non-classical inertia in superconductors, eq.(29):

$$|g_0| = \frac{\chi}{2} |A| \quad (38)$$

Thus our hypothesis is that the strong applied electric field in a Tao cell triggers the creation of gravitational fields that are much stronger than usual, via a suitable scale transformation $G \rightarrow G'$. From Newton's law we calculate the Tao ball acceleration as

$$|A| = \frac{|q|E_0}{m_{TB}}, \quad (39)$$

where m_{TB} is the Tao ball mass. The electric charge $|q|$ acquired by a Tao ball while bouncing between the electrodes is given by [13]

$$|q| = 4\pi\epsilon_0 k_L E_0 a^2, \quad (40)$$

where a is the radius of the Tao ball, $k_L = 1.44$ is the dielectric constant of liquid nitrogen, and ϵ_0 is the vacuum permittivity in SI units. Substituting equations (38), (39) and (40) into eq.(37) we obtain the gravitational surface tension of a Tao ball as

$$\sigma_g = \frac{\chi}{2} k_L \epsilon_0 E_0^2 a. \quad (41)$$

Substituting the law defining χ , eq.(30), into eq.(41), we see that this gravitational surface tension is proportional to the fourth power of the critical transition temperature T_c of the superconductive material:

$$\sigma_g = \frac{3}{2} \frac{(\ln 3)^4}{4\pi} \left(\frac{T_c}{T_{PE}} \right)^4 k_L \epsilon_0 E_0^2 a \quad (42)$$

As T_c increases the gravitational surface tension increases. This is in qualitative agreement with the experimental observation: According to [14], the

Tao balls formed by low- T_c superconductors are weaker and easier to break than those formed by high- T_c superconductors.

NdBCO high- T_c superconductors have a critical temperature of $T_c = 94K$. In that case the numerical prefactor in eq. (42) reduces to 1, and our theory for σ_g then reproduces the same result that was derived in [13] for the surface tension using a different model:

$$\sigma_g^{NdBCO} = k_L \epsilon_0 E_0^2 a \sim 2 \times 10^{-3} [N/m] \quad (43)$$

It is interesting to note that if we substitute the spacetime volume of a Tao ball, $V = \frac{4}{3}\pi a^3 c\Delta t$, into the space-time uncertainty relation in a superconducting environment, eq.(31), we find the following expression for the typical radius of a Tao ball:

$$a \sim \pi \left(\frac{1}{3\chi^2} \frac{l_{PE}^4}{c\Delta t} \right)^{1/3} \quad (44)$$

Assuming that the temporal length $c\Delta t$ in eq.(44) is equal to the size $c\Delta t \sim 1\mu m$ of the microparticles forming the Tao ball, we obtain the correct order of magnitude for the radius of Tao balls, i.e., $a \sim 0.17[mm]$ for the case of NdBCO (with $\chi^{NdBCO} = 2$). This means that the coarse-grained microstructure of Tao balls consisting of many smaller particles of μm size is correctly described. Tao's experiments could be seen as the spatial counterpart of the experiments with radioactive superconductors discussed above, which deal with the temporal aspects of the space-time volume fluctuations.

6 Further experimental suggestions

The scale transformation of section 4 strongly enhances gravitational effects in a superconducting environment. At the same time, it strongly suppresses unwanted vacuum energy. Effects induced by the scale transformation should be measurable. A couple of interesting laboratory tests can then be performed with superconductors. In the following, we list a few proposals in this direction:

1. Measuring the cutoff frequency of quantum noise spectra in superconductors. This experiment is currently performed in London and Cambridge (UK), extending previous work of Koch et al. [39].
2. Measuring gravitomagnetic fields and frame dragging effects in the vicinity of rotating superconductors. These experiments are currently

performed in Seibersdorf (Austria) and Canterbury (NZ) [26, 40]. Performing similar experiments with rotating supersolids would also be a valuable concept [41].

3. Measuring time with high precision inside and outside superconductive cavities. Since some inertial-like effects of rotating superconductive rings seem to propagate outside the ring [24, 25, 26], it would also be interesting to probe for temporal statistical fluctuations in the neighborhood of a superconductive material. In order to carry out this investigation we propose to compare the measurement of time intervals measured by two synchronized identical clocks located inside and outside a superconductive cavity. The clocks would be synchronized before the cavity is made superconductive, and at constant intervals of time, the time indicated by the clocks would be regularly compared, in order to probe for any difference between time statistical fluctuations inside and outside the cavity.
4. Investigating broadening phenomena of the decay energy line width in radioactive superconductors.
5. Measuring Coriolis forces on test masses moving inside rotating superconductive cavities. This would basically correspond to carrying out Foucault-type pendulum experiments inside rotating superconductive cavities.
6. Comparing the measurement of acceleration exerted on masses inside accelerated superconductive cavities with similar accelerations detected inside cavities made of normal materials.
7. Carrying out the famous Einstein *Gedanken* elevator experiment, by comparing the measurement of the acceleration inside a superconductive cavity falling under the sole influence of the earth's gravitational field with that of a cavity made of normal materials, which would also be in a state of free fall.
8. Repeating the small-scale tests of the gravitational inverse square law as performed by Adelberger et al. [42, 43] in a superconducting environment.
9. Investigating the formation of Tao balls [13] in more detail. Is their formation connected with anomalous acceleration and gyroscope signals? Place accelerometers and gyroscopes near to the Tao cell, similar as in Tajmar's experiments [25].

10. Investigating a rotating Tao cell. How do Tao balls form in a rotating environment? Compare with planetary aggregation models where G is replaced by a rescaled G' . Similar questions could be dealt with when a magnetic field is applied to the Tao cell [14, 15].
11. Carrying out Hertz-like experiments with Tao balls and checking for gravitational radiation, in line with a similar proposal of Chiao[44] for electrically charged superfluid Helium droplets.
12. Performing high-precision measurements of force fields in superconductors using SQUIDS and Josephson junction arrays, in line with a suggestion of Fischer et al. [45]

7 Conclusion

In this paper we were turning the cosmological constant problem around, to argue that there is also an inverse cosmological constant problem where formally the cosmological constant comes out 120 orders of magnitude *too small*. For the inverse cosmological constant problem, one starts from the Einstein Hilbert action and considers an uncertainty relation for 4-dimensional space-time. The true value of the cosmological constant, as observed by WMAP, is given by the geometric mean of both approaches, the quantum field theoretical one predicting a value 120 orders of magnitude too large and the one starting from the Einstein-Hilbert action, predicting a value 120 orders of magnitude too small. This intermediate value represents the Planck-Einstein scale, a natural scale for dark energy, superconductors, and solid state physics in general.

We have formulated the hypothesis that in a superconducting environment it makes sense to formally consider a scale transformation from the Planck scale by the Planck-Einstein scale. This scale transformation leads to a strong suppression of certain quantum mechanical observables (such as vacuum energy density) and strong enhancement of others (such as gravitomagnetic fields). We have shown that these suppression and enhancement effects are consistent with some recent experimental observations. The formation of large superconducting spherical balls (the Tao effect) can also be understood in this context. A fundamental space-time uncertainty in a superconducting environment is predicted, which can be checked by future experiments. Ultimately non-classical inertia in superconductive cavities can be related to fluctuations in the number of relevant Planck-Einstein sized space-time cells in a superconductor. We believe that it is important to perform further precision experiments with superconductors, to fully explore the

Planck-Einstein scale and to further investigate the connection between dark energy, gravity, and the physics of superconductors.

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